Astronomy Mission

Introduction

Mathematics has many far-reaching applications, with some reaching as far as the stars. In fact, astronomy is one of the oldest sciences and has helped us to further our understanding of the universe. Astronomy is more than just creating a map of the observable stars, it is an important field of study which focuses on the building blocks of our universe. Fundamental mathematical techniques, such as trigonometry, enable us to uncover important astronomical phenomena and planetary information, including distances to celestial objects, orbital speeds, and even the mysterious force of gravity. This workshop will therefore provide students with the opportunity to use the mathematics they are already familiar with to study a new planet orbiting a distant star.

Aim of Workshop

The aim of this workshop is to show students the applicability of mathematical concepts, particularly in the realm of the natural sciences. Through working in groups to solve the problems arising from planning an interstellar mission, it is hoped that students will experience the collaborative nature that is central to STEM-based work. Moreover, this workshop aims to reintroduce mathematical topics such as scientific notation and trigonometry in an astronomical context, whilst also exploring new concepts such as force, parallax, and Kepler's laws of planetary motion. (This workshop may be particularly relevant for students interested in Physics and/or Applied Mathematics).

Learning Outcomes

By the end of this workshop students will be able to:

- Explain, in their own words, what is meant by parallax
- Convert large numbers such as distances/ speeds to scientific notation
- Apply trigonometric ratios to find distances between planets and stars

Materials and Resources

Scale of the universe video clip (optional)

Keywords

Period
A period of motion is the length of time it takes a planet to complete a cycle of revolution about the Sun

Arc second
An arc second is 1/3600th of a degree. Just as there are 60 minutes in an hour and 60 seconds in a minute, a degree is divided into 60 arc minutes and an arc minute is divided into 60 arc seconds

Astronomical Unit (AU)
Unit of length used by astronomers for distances in space. One astronomical unit is equal to the average distance between the Earth and the Sun (approx. 150 million kilometres)

Astronomy Mission: Workshop Outline

SUGGESTED TIME (TOTAL MINS)	ACTIVITY	DESCRIPTION OF CONTENT
5 mins (00:05)	**Introduction to Astronomy**	– Mention the importance of mathematics in astronomy, particularly trigonometry (see **Workshop Introduction**). – Outline the aim of the workshop (to discover information about a new planet).
5 mins (00:10)	**Trigonometry Revision**	– You may wish to revise the trigonometric ratios sine, cosine, and tangent for right angle triangles.
5 mins (00:15)	**Parallax**	– Explain what is meant by parallax and ask students to try the Blink Test (see **Appendix – Note 1**). – Ask them what they noticed? Which finger appeared to move the most? – Class discussion on how parallax might help us determine how far away a star/planet is.
10 mins (00:25)	**Scale of the Universe and Scientific Notation**	– Brief video on the scale of the universe (see link in **Additional Resources**). – Mention the need for special notation to represent such large (and small) numbers. – You may wish to revise scientific notation with students (see **Appendix – Note 2**).
15 mins (00:40)	**Activity 1 The Expedition Begins**	– Hand out the formulae sheet to each pair of students. – **Activity Sheet 1**: In pairs, students calculate the distance to a new planet (see **Appendix – Note 3**).

SUGGESTED TIME (TOTAL MINS)	ACTIVITY	DESCRIPTION OF CONTENT
5 mins (00:45)	**Revision**	– Describe what is meant by force and mention Newton's second law of motion (see **Appendix – Note 4**). – Discuss the difference between weight and mass and ask students if they know the units for each (see **Appendix – Note 4**). – You may wish to recap the distance-speed-time relationships. – Emphasise the importance of distance and velocity having the same units e.g. **km** and **km/hr** or **m** and **m/sec** respectively.
10-15 mins (01:00)	**Activity 2 The New Planet**	– **Activity Sheet 2:** In pairs, students now attempt activity 2 using the formulae sheet (see **Appendix – Note 5**).
5 mins (01:05)	**Kepler's Laws**	– Discuss Kepler's laws of planetary motion and mention Newton's contributions (see **Appendix – Note 6**).
10 - 15 mins (01:20)	**Activity 3 The New Planet**	– **Activity Sheet 3**: Students now try to find the period of the new planet (see **Appendix – Note 7**). – Ask students if they noticed anything particular about the period of this new planet.

Astronomy Mission: Workshop Appendix

Note 1: The Blink Test and Parallax

Parallax is the apparent shift in the position of an object due to a change in the observer's point of view. The effect can be demonstrated by closing one eye and holding up your index finger at arm's length. Place your other index finger at a position that is closer to your face (see Figure 1). Now, look at your fingers with the other eye. As you alternate between your left and right eye, your fingers appear to move back and forth. This observation is due to the fact that the position from which you are viewing your finger is changing; a phenomenon known as parallax. Notice also that the finger closest to you appears to move the most relative to the background. When objects are too far away however, this shift in position is not apparent. This serves to demonstrate the limitations of approximating distances by parallax.

A further illustration of parallax (or triangulation) is shown in Figure 2 below. In this case, the tree is to the right of Person A and to the left of the Person B due to their different points of view. We can calculate how far away the tree is by using the angle of parallax and basic trigonometric calculations.

Figure 6: Blink test demonstration

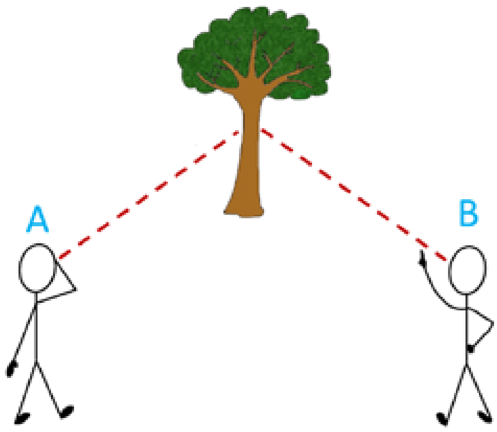


Figure 7: Example of Parallax

This same effect occurs when observing (presumed stationary) stars from earth as it orbits around the Sun. Some stars appear to move a lot, whilst others do not seem to move as much. Therefore, by measuring the parallax angle, it is possible to determine approximately how far away a celestial object is. Astronomers achieve this by viewing the star or planet from two different locations in the Earth's orbit around the Sun. For very distant stars, there does not appear to be any shift in position and consequently, it can be difficult to estimate their distance from the Earth.

Note 2: Scientific Notation

Formula for scientific notation: (number between 1 and 9) x 10^{power}

Example questions to ask students:

Q1. The closest distance between Earth and Pluto is 4,300,000,000 kilometres. How could we write this in scientific notation?

4.3×10^9 kilometres

Q2. What would 5326.6 be in correct scientific notation?

5.3266×10^3

Note 3: Solutions for Activity 1

Q1. Using your knowledge of trigonometry, find the distance from Proxima B to where the signal is coming from (blue planet).

We can bisect the parallax angle to form two right angled triangles.

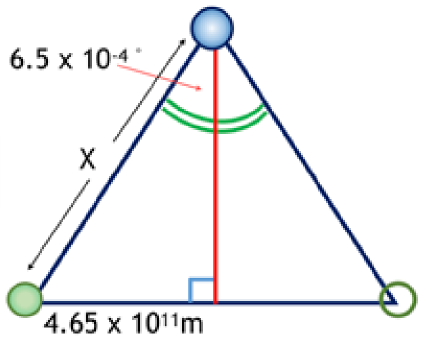
The angle at the top vertex is therefore:

$1.3 \times 10^{-3}/2 = 6.5 \times 10^{-4}\,°$

To find the desired distance x we can use: $\sin\theta = \dfrac{opposite}{hypothenuse}$

$$\sin(6.5 \times 10^{-4}) = \frac{(4.65 \times 10^{11})}{x}$$

$$x = \frac{(4.65 \times 10^{11})}{\sin(6.5 \times 10^{-4})} = 4.09885 \times 10^{16}$$



Q2. You are travelling at a speed of 1.5×10^8m/s (half the speed of light), how long will it take you to reach the new planet in seconds? (Hint: how are distance, speed and time related?)

Time = Distance/Speed
$T = 4.09885 \times 10^{16}/ 1.5 \times 10^8 = 273256794.6$ seconds

(Note: this answer may vary depending on the number of decimal places that are kept from the previous answer. The above answer was computed by using the ANS function on the calculator)

Q3. Can you calculate how many years it will take to reach the new planet?

60 secs in a min
60 mins in an hour
24 hours in a day
365 days in a year

60 x 60 x 24 x 365 = 315360000 seconds in a year
No. of years = 273256794.6/315360000
No. of years = 8.66 years

Note 4: Force, Mass and Weight

Force

Force is any interaction that, when unopposed, causes a change in the motion of an object. This idea is summarised by **Sir Isaac Newton** in his Laws of Motion. In fact, Newton's second law describes force as the mass of an object times its acceleration, more commonly denoted by the formula, F = ma. The SI unit for force is Newton (N), with 1 Newton equal to 1 kg x m/s^2.

Mass and Weight

The mass of an object is a measure of the amount of matter it contains, whereas its weight is the gravitational force that is exerted on it. Since an object or body will have the same composition regardless of its location, mass is independent of gravitational forces. An individual with a mass of 60kg on Earth, for example, would also have a mass of 60kg on the moon, given that they still contain the same amount of matter! Weight, on the other hand, is dependent on gravity and consequently, this same individual would weigh less on the moon since the force of gravity is far less on the moon than on Earth. In fact, weight is defined as the mass of an object times its acceleration due to gravity and since weight is a force, the SI unit is Newton (N).

Note 5: Solutions for Activity 2

Q1. What is the value of g on Planet Nua?

W = mg	g = 784/80
g = W/m	$g = 9.8 m/s^2$

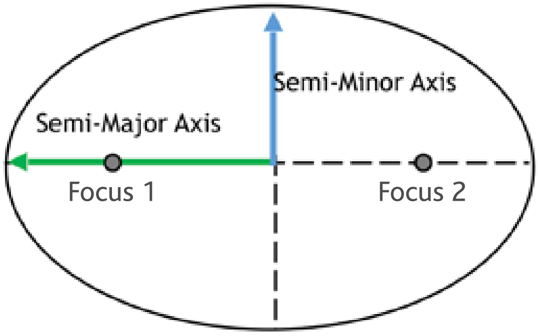
Q2. Calculate Newton's Gravitational constant (G).

$$G = \frac{gR^2}{M} = \frac{(7.15)\,(5.1\times10^7)^2}{2.79\times10^{26}} = 6.67 \times 10^{11}\ m^3\ kg^{-1}\ s^{-2}$$

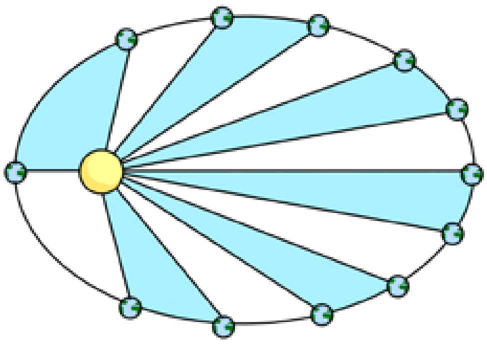
Note 6: Kepler’s Laws of Planetary Motion

Johannes Kepler was a German mathematician and astronomer who devised three laws to describe the motion of planets orbiting the Sun, each of which is outlined below.

1. All planets move in elliptical orbits about the Sun. An ellipse is a regular oval shape, characterised by two foci, a minor axis, and a major axis. The sun is located at one of the two foci.



2. An imaginary line from the centre of the Sun to the centre of a planet will sweep out equal areas in equal intervals of time (see diagram below). The closer a planet is to the Sun, the stronger the Sun’s gravitational pull and thus, the faster the planet’s orbital speed.



3. Kepler’s third law describes the relationship between a planet’s distance from the Sun and the time it takes to complete its orbit (period). The larger the orbit, the longer it will take the planet to complete it given that the Sun’s gravitational pull will be weaker when it is further away. This law is represented by the following formula:

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

Where **T** is the period, **a** is the semi-major axis, **M** is the mass of the planet, **G** is Newton’s gravitational constant and π is our old constant friend pi.

Whilst Kepler described the motions of the planets, he did not provide any explanation of why the planets move in this way. In fact, Kepler's third law only applies to planets that orbit around the Sun but not other orbits such as the Moon's orbit around the Earth, for example. However, Sir Isaac Newton provided a more general explanation for the motions of the planets and other celestial objects, which is summarised in his Universal Law of Gravitation and Laws of Motion.

Note 7: Solutions for Activity 3

Q1. Assuming the phone signal is fibre optic (travels at the speed of light), calculate the distance between Planet Nua and Planet Eile (The value for the speed of light is on the formulae sheet).

3 minutes = 180 seconds. Using the distance formula, we get:

Distance = speed x time = $(3 \times 10^8 m/s)(180s) = 5.4 \times 10^{10}m$

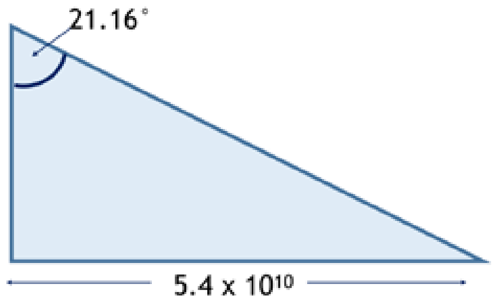
Q2. Another research team have found the angle between the vertices of Planet Nua, the star, and Planet Eile to be 21.16°. Find the distance from Planet Nua to the star (i.e. the semi-major axis).

We can use the formula:

$$\text{Sin}\theta = \frac{opposite}{hypothenuse}$$

$$x = \frac{5.4 \times 10^{10}}{\sin(21.16)}$$

$x = 1.49595 \times 10^{11}m$



Q3. Using the information you have calculated, find a value for the period of Planet Nua and change it from seconds to days. Do you notice anything significant about this figure? (Take M = 1.9891 × 10³⁰ kilogrammes for the mass).

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

$a = 1.49595 \times 10^{11}m$
$G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$ (same for home planet)
$M = 1.9891 \times 10^{30}$

Substituting this into the formula above we get:

$T^2 = 9.96 \times 10^{14}$
$T = 31559467.68$
$T = 31559467.68/(60 \times 60 \times 24)$ days
T = 365.3 days...i.e. 1 year. The mystery planet is Earth!

Sources and Additional Resources

https://www.youtube.com/watch?v=uaGEjrADGPA (Scale of universe video)
http://www.physicsclassroom.com/class/circles/Lesson-4/Kepler-s-Three-Laws (Kepler's laws)

Astronomy Mission: Formulae Sheet

Distance	distance = speed x time
Newton's Law of Universal Gravitation	$F = \frac{GMm}{R^2}$
Gravitational Constant	$G = \frac{gR^2}{M}$
Force (Newton's Second Law)	F = ma
Weight	W = mg
Kepler's Third Law	$T^2 = \frac{4\pi^2 a^3}{GM}$

Note:

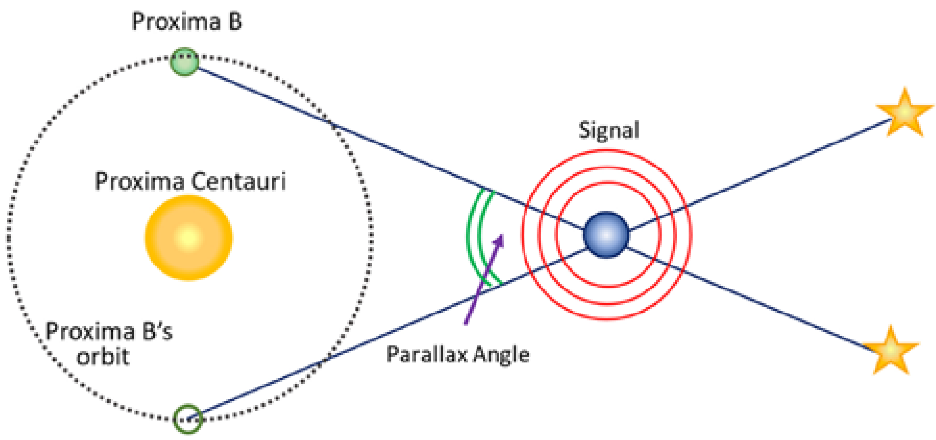
g = Acceleration due to gravity
M = Mass of planet
R = Radius of planet
G = Newton's gravitational constant
F = Force
T = Period
V = Velocity
a = Semi-major axis

Speed of Light C = 3 x 10^8 m/s

Astronomy Mission: Activity 1

The Expedition Begins

You are on planet Proxima B in an orbit around the star Proxima Centauri as shown below. The star Proxima Centauri is approximately 3.1 Astronomical Units (4.65×10^{11}m) from the centre of your planet. A signal has been detected from outer space, possibly from a planet which was, until now, undetected. Through methods of parallax, a research team have found that the Parallax Angle from the point of origin of the signal is 4.68 arc seconds
(or 1.3×10^{-3} degrees).



Q1. Using your knowledge of trigonometry, find the distance from Proxima B to where the signal is coming from (Blue planet). (Draw your blueprints)

Q2. You are travelling at a speed of 1.5×10^8m/s (half the speed of light), how long will it take you to reach the new planet in seconds? (Hint: how are distance, speed and time related?)

Q3. Can you calculate how many years it will take to reach the new planet?

Astronomy Mission: Activity 2

The New Planet

Congratulations you have discovered a new planet, which we will call Planet Nua!

You would like to find out how strong the force of gravity is on Planet Nua. Suppose your mass is 80kg. You step on a scale on the new planet and your weight reads 784N.

Q1. What is the value of g on Planet Nua?

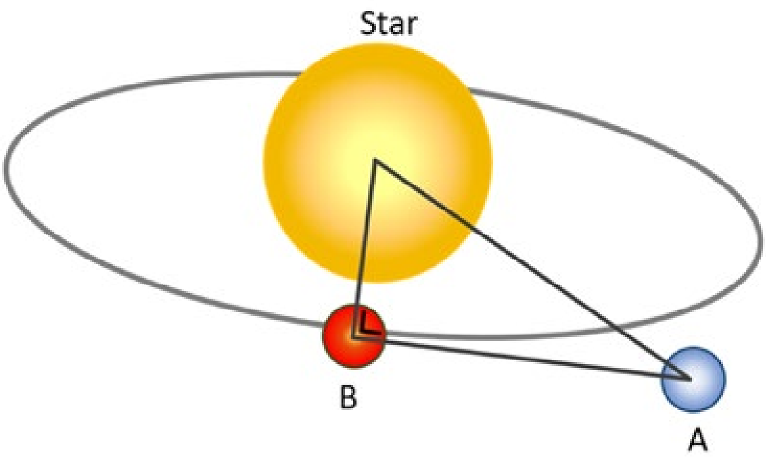
Q2. Taking values from your home planet to be

- $g = 7.15 m/s^2$
- $M = 2.79 \times 10^{26} kg$
- $R = 5.1 \times 10^7 m$

Calculate Newton's Gravitational constant (G). Note: G and g are different!

Astronomy Mission: Activity 3

You are on Planet Nua (A). You would like to know the distance from your planet to the nearest star. Your friends are chilling on Planet Eile (B) so you call them up. The time delay between you calling and your friends receiving the call is 3 minutes.



Q1. Assuming the phone signal is fibre optic (travels at the speed of light), calculate the distance between Planet Nua and Planet Eile (The value for the speed of light is on the formulae sheet).

Q2. **Another research team have found the angle between the vertices of Planet Nua, the star, and Planet Eile to be 21.16°. Find the distance from Planet Nua to the star (i.e. the semi-major axis).**

Q3. **Using the information you have calculated, find a value for the period of Planet Nua and change it from seconds to days. Do you notice anything significant about this figure?**

(Take $M = 1.9891 \times 10^{30}$ kilogrammes for the mass).